Liquid-In-Solid Convection

by

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Abstract

A model is proposed in which upward-moving, liquid-in-solid convection cells are the primary mechanism by which the Earth's interior is cooled. Convection cells form in the liquid core when heat from radioactive sources is convected outward. Each cell penetrates the mantle by melting the rock above it due to the extra heat transported by the cell. In this way each convection cell propagates upwards through the otherwise solid mantle as the convected heat melts rock above the cell and solidifies magma below it. Because the cells are initiated in the core spontaneously and at random, the model accounts for the observed random nature of many geophysical phenomena. A laboratory experiment is proposed whereby a low melting point solid such as wax is subjected to a high temperature gradient in order to promote liquid-in-solid convection and so emulate aspects of mantle dynamics.

Introduction

Spectral analysis of observational data by Pelletier (2002) indicates that the geomagnetic field has a power law variance spectrum with no sharp peaks that would indicate deterministic, periodic behaviour. The observed random occurrence of geomagnetic reversals is described by such a power law spectrum. A more profound implication is that processes which dominate the Earth's interior are primarily random or stochastic, i.e. governed by the laws of probability. Geothermal processes are evidently not steady-state and cannot be accounted for by deterministic models.

Conventionally the mantle is assumed to be sufficiently plastic to allow convection but sufficiently rigid to support seismic S-waves; see, for example Turcotte and Oxburgh (1972) and Richter (1984) and references therein. The problem is that such plastic mantle convection is very slow, with turnover times of the order of 1×10^9 years, whereas Pelletier's spectrum follows its power law locus to periods shorter than 1000 years.

It might be argued that geomagnetism is generated entirely in the core – for example, the deterministic model of Glatzmaier and Roberts (1995). This implies a spectrum which is independent of mantle dynamics. Even so questions concerning the random nature of geomagnetism and the relatively short time scales of mantle processes remain unresolved.

A simple, one dimensional model in which conduction is the only form of heat transport predicts a fluid core with a size and temperature far in excess of those deduced from observation. For instance assuming uniform radiogenic heating, $H_r = 1.4 \times 10^{-8}$ W m^{-3} (Pollack et al., 1993), and uniform conductivity ($\kappa = 1.0$ W.m^{-1}K^{-1}) (Tang et al., 2014) gives a core temperature of $27,000\,^{\circ}$C and a core radius of 6100 km. In the present epoch the liquid core radius is observed to be only 3500 km, and so conduction alone cannot account for the observed structure of the Earth's interior. It follows that some mechanism of heat transport other than conduction and visco-plastic turnover must play a major role in cooling the Earth.

Liquid-in-solid convection

We propose an alternative, as depicted in idealized form in Figure 1. In Figure 1, convection cells form in the liquid outer core. Rayleigh–Bernard convection is a stochastic process. It occurs when a body of liquid lies in a vertical temperature gradient at length scales sufficiently large for the critical Rayleigh number to be exceeded. As a consequence of the formation of these cells, regions of the lower mantle, M, which are directly above each

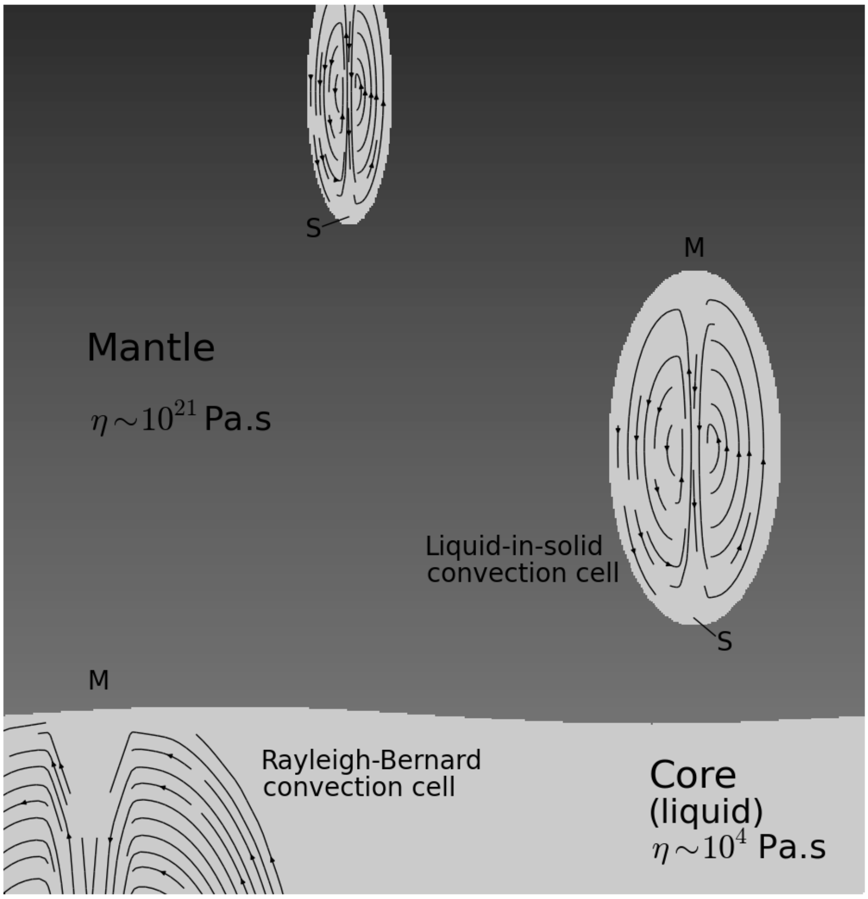

Figure 1: A conceptualization of liquid-in-solid convection cells formed in the lower mantle from Rayleigh–Bernard convection cells in the core. M: regions of melting due to increasing temperature. S: regions of solidification due to cooling and increasing pressure. Note that this diagram is highly idealized and does not represent numerical model output. The approximate dynamical viscosities, η, are also shown.

Table 1: Values of the Rayleigh Number, R_a, as a function of length scale and viscosity.

L km	η =1 Pa.s	η =1×10^{11} Pa.s
0.001	10×10^{-3}	10×10^{-14}
0.010	10×10^{1}	10×10^{-10}
0.100	10×10^{5}	10×10^{-6}
1.000	10×10^{9}	10×10^{-2}
10.000	10×10^{13}	10×10^{2}
100.000	10×10^{17}	10×10^{6}
1000.000	10×10^{21}	10×10^{10}

Rayleigh–Bernard convection cells will attain a higher temperature than surrounding regions because of the extra heat transported by the cell. These regions then melt allowing the cell to penetrate the mantle. Ultimately the cell becomes completely enclosed by the solid mantle as the mantle solidifies behind it in region S. This happens because the increased heat transport promoted by the cell causes the mantle below it to be cooled.

This process will be designated LISC, for "liquid-in-solid convection". Rayleigh number calculations indicate that such LISC cells are feasible. Their existence or otherwise depends on whether the Rayleigh number, R_a, for such a cell exceeds the critical Rayleigh number, R_{ac}. This in turn depends largely on their scale size, viscosity and other parameters and is given by

$$R_a = \frac{\rho\alpha(T_{bottom} - T_{top})gL^3C_p}{\eta k} \tag{1}$$

where ρ is the density, α is the coefficient of thermal expansion, T is the temperature, L is the length scale, C_p is the heat capacity, η is the viscosity, and k is the thermal conductivity. This can be written

$$R_a = \frac{\rho\alpha\Delta T gL^4C_p}{\eta k} \tag{2}$$

where ΔT is the temperature gradient.

One of the factors in (2), the viscosity, η, is not well known. Estimates of its value in the lower mantle vary from 1 to 1×10^{11} Pa.s. A similar variation is found in near-surface magmas depending on their composition. Despite this, meaningful values of R_a can be found because it is such a strong function of L in (2). Table 1 shows some values at different length scales for both low and high viscosity values.

Convection can only occur when R_a exceeds the critical Rayleigh Number, R_{ac}. A nominal value for the latter is 1707, but it varies slightly according to the geometry. Hence, according to Table 1, the length scale above which LISC is possible lies somewhere between 300 m and 30 km depending on the assumed viscosity.

Each cell melts the solid mantle above it because of the extra heat being convected outward from the hot core. In this way each convection cell propagates upwards through the otherwise solid mantle at speeds determined by the solution of the Stefan problem[1] for a liquid-solid boundary. The upward-moving, LISC cells formed in this way are proposed as the primary mechanism by which the interior of the Earth is cooled.

The LISC cells shown in Figure 1 are highly idealised. In reality LISC cells are unlikely to be neat ellipsoids. The solution of the Stefan problem for a moving phase transition within a non-uniform medium will be difficult or intractable, but intuitively it seems likely that rising cells will follow paths where the melting point is lower or the solid is hotter and avoid places where it is cooler. Thus strings of ascending LISC cells can be expected to

[1]The Stefan problem is the mathematical description of a phase boundary which moves with time.

form wherein each cell follows the path of a preceding cell. Such a string of LISC cells could well be described as a "plume" when it is in the mantle. An individual LISC cell becomes a "diaper" when it interacts with the crust.

Some implications of a random model

The initial generation of convection cells is a random process. It is impossible to predict when or where a new cell will start up or how big it will be because turbulent convection itself is a random process. All physical quantities associated with LISC will therefore also have a random component and have power law spectra. These include ascent speed, volume, magnetic field and temperature.

In this model of the Earth's interior, specific heat, thermal conductivity, radiogenic heating and density are all assumed to be constant. Despite this simplicity, complex behaviour occurs as a consequence of convection. Rayleigh–Bernard convection cells form spontaneously and at random in the outer liquid core as heat builds up from radioactive sources. The model assumes that the primary distinction between mantle and core is due to phase alone: one is solid, the other liquid. (This does preclude compositional differences. The concentration of heavier elements such as iron can be expected to be greater nearer the centre of the Earth.)

- LISC cells will have a large range of scales, ranging from the Rayleigh scale to scales comparable to the dimensions of the core itself, from volcanic eruptions to Large Igneous Provinces.
- The total volume of the Earth is the sum of the volumes of the core, the solid mantle and all the LISC cells in the mantle. It must also be a quantity which varies over time in a random manner according to the proportion of molten material in the core and mantle.
- Hence sometimes the earth expands and at other times it contracts. When the Earth is expanding, new crust forms at mid-ocean ridges (Fig. 5)
- When the Earth contracts, the crust is forced to crumple and mountain building occurs. Such cooling and crumpling would result from, and be modified by, extensive diapirism.
- Sea floor spreading occurs when the Earth expands – recent magnetic stripes average 6 km across indicating a total expansion of $\sim$36 km in the Earth's circumference, i.e. roughly 0.1 percent variation in circumference during each expansion phase.
- The geomagnetic field is the sum of the self-exciting magneto-hydrodynamic (MHD) dynamo fields of all of the LISC and Rayleigh–Bernard convection cells in the mantle and core respectively. It is therefore a random quantity which varies over time in a random manner.
- Glatzmaier and Roberts (1995) account for isolated field reversals but not the variance spectrum of the geomagnetic field observed by Pelletier. Their deterministic model could be upgraded to include random forcing as described here.

A wax block experiment

Numerical modeling is deterministic and so cannot fully emulate the random aspects of convection. However, modelling of individual LISC cells under various viscosity and temperature gradient scenarios could provide important insights. In particular, local solutions of the Stefan Problem under physical conditions pertaining in the mantle could be used to provide realistic parametrizations for use in larger scale models.

However, the reality of liquid-in-solid convection should first be verified at laboratory scale. This can be done, for example, by subjecting a block of wax to a vertical temperature gradient. Such an experiment could provide great insight into plume formation and diaperism.

Once again, the relevant parameter is the Rayleigh Number. For, say, a ~ 1 m^3 block of paraffin wax on an electric hot plate at, say, 100 deg C, cooled at the top with iced water, and assuming $g = 9.8$ m.s^{-2}, $k = 0.25$ W.m^{-1}K^{-1}, $C_p = 2500$ JK^{-1}.kg^{-1}, $\alpha = .0007$ K^{-1}, $\rho = 900$ kg.m^{-2} $\eta = 3.0$ Pa.s, $\Delta T = 100.0$ K, $R = 1707$, substituting into (2) and solving for x gives a critical Rayleigh scale of $x \approx .01$ m. Thus molten LISC cells should form at scales of 1 cm or more in solid wax under these conditions.

An interesting aspect of this experiment is whether permanent liquid layers would form both at the base and just below the cold top crust, so corresponding to the core and asthenosphere of the Earth. If so, the interaction of ascending LISC cells with the asthenosphere and crust layers might emulate some of the effects of diaperism discussed by Carey (1999).

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